ENERGY BALANCE IN THE DISCHARGE CIRCUIT OF AN ELECTRIC-DISCHARGE LASER

B. A. Barikhin^{*)} and Yu. I. Kryshalovich

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The possibility of using transient processes in the discharge circuit of an electric-discharge laser to pump the active media is analyzed. It is shown that, due to the nonlinearity of the transient process, under certain conditions, the efficiency of the energy contribution to the alternate load can almost be doubled as compared with the circuit containing a permanent load. It has been established that the process is quasiperiodic when the discharge-plasma resistance changes by an exponential law. The conditions of realization of the maximum energy contribution as well as the regions of stabilization of the optimum parameters have been determined. The necessary calculations have been performed for active media that model solutions of organic dyes under excitation by flashlamps.

Generated radiation losses in a dye laser have been studied in sufficient detail [1-4]. However, the high energy output of organic-dye lasers is provided predominantly by the high energy release in the pumping source. Because of this, the pumping source must meet a number of exacting requirements, the main of which, including alternative requirements, can be determined at the qualitative-analysis level. This does not mean that there is no way of performing the corresponding quantitative estimations and calculations – they will add nothing to the results of the qualitative analysis. At the same time, the efficiency of qualitative estimates is apparent and makes it possible to work out concrete recommendations, for designing pumping sources for organic-dye lasers with a high output energy.

A conventional electric-discharge laser represents a discharge circuit containing an electric (magnetic) energy storage, a controlled high-voltage commutator, a current line, and a resistive load that are connected in series. The main functional requirement imposed on the power source of the laser module is the possibility of maneuvering rapidly the portion of the energy stored in the energy storage, which is released in the load, and the pulse duration, within which this energy is released. In this case, account must be taken of the special properties of the process of excitation in organic-dye lasers that manifest themselves in an increase in the induced losses in the active medium of these lasers incomparably larger than that in other lasers, which can cause the suppression of oscillations even at the leading edge of the pumping pulse [4]. Because of this, for organic-dye lasers, as distinguished from other lasers, of importance is not the value of the pumping energy integral over the pulse but its portion that is released during the leading edge of the pumping pulse. Consequently, the problem of optimization of the parameters of the power source of an organic-dye laser is the achievement, first, of the maximum energy released in the load at the leading edge of the pumping pulse and, second, of the minimum duration of the leading edge. For a bank of capacitors, the pulse power is determined by the working voltage U_0 of the power source and the electric capacity C_e of the storage. In the discharge circuit, the pulse duration can be changed in two ways: through the sectionalization of the capacity storage and by changing the wave resistance of the current line in each section. Thus, the energy $W_e = C_e U_0^2/2$ can be stored in the capacity storage. The condition of the maximum release of the stored energy in the load is provided by the balance of the electrical resistance of the load R_c and the wave resistance of the circuit $\rho_c =$ $\sqrt{L_c/C_c}$. Usually, for fast circuits with large values of the stored energy [5, 6] the active resistance of the

^{*)} Deceased.

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storage $R_e \ll R_c$, the capacity of the circuit $C_c \ll C_e$, and the inductances of the storage L_e and of the discharge circuit L_c are of the same order.

A discharge of the capacitor bank in the resistive load represents a transient process in the discharge circuit, which is described by the equation

$$\frac{d^2 u_{\text{cond}}}{dx^2} + D_c(x) \frac{du_{\text{cond}}}{dx} + u_{\text{cond}} = 0, \qquad (1)$$

where $u_{cond} = U_{cond}(t)/U_0$ is the dimensionless voltage across the capacitor bank expressed in terms of the instantaneous U(t) and working U_0 voltages; $x = \omega_0 t$ is the dimensionless time expressed in terms of the frequency of oscillations of the unloaded circuit $\omega_0 = (L_c C_c)^{-1/2}$; $D_c(x) = R_c(t) = R_c(t)/\rho_c$ is the discharge-circuit damping expressed in terms of the variable electrical resistance of the load $R_c(t)$ and the wave resistance of the circuit ρ_c . The Q-factor of the discharge circuit is related to the damping by the known relation $Q_c = 1/D_c$. For circuits with a steady damping it is convenient to introduce the notation $d = R_c/2\rho_c = \text{const}$ and to write Eq. (1) in the form

$$\frac{d^2 u_{\text{cond}}}{dx^2} + 2d \frac{du_{\text{cond}}}{dx} + u_{\text{cond}} = 0.$$
⁽²⁾

It is more informative to perform analysis of the energy balance in the transient process with the use of dimensionless quantities:

a) strength of current in the discharge circuit $J_c(x) = du_{cond}(x)/dx = i_c(t)/U_0/\rho_c$ expressed in terms of the dimensional quantity $i_c(t)$;

b) instantaneous power of the heat release in the active resistance of the load

$$P_{\rm c}(x) = D_{\rm c}(x) \left(\frac{du_{\rm cond}}{dx}\right)^2 = \frac{R_{\rm c}(t) i_{\rm c}^2(t)}{U_0^2 / \rho_{\rm c}};$$

c) energy released in the form of heat in the active resistance by the time t, $w_c(x) = 2\int_0^x D_c(y) \left(\frac{du_{cond}}{dy}\right)^2 dy = \int_0^t R_c(t) i_c^2(t) dt / C_e U_0^2 / 2$ expressed in terms of the energy stored in the storage W_e .

The properties of the analytical solution of Eq. (2) are well known, in particular, the energy release in the load, integral over the pulse duration, attains the maximum value and is performed in a minimum time interval under the critical discharge condition d = 1. In designing power sources for solid-state lasers, the steady-state condition of an aperiodic process is usually taken as the operating condition, since to pump these lasers it is necessary to form a discharge plasma with a low brightness temperature $T_b \leq 5 \cdot 10^3$ K in pulses of duration of ~1 µsec that have a nearly rectangular shape [7]. In this case, the condition $d \geq 10$ must be fulfilled with good accuracy. However, the analogous requirement imposed on the pumping source of an organic-dye laser is different and is determined by the nonlinearity of the driving-pulse leading-edge rise. In this connection, the energy release in the load, integral over the time of leading-edge rise to the maximum $x_m = 1.24$, attains the maximum value $w_m = 0.40$ of the stored energy at the maximum value of the discharge-current strength $i_m = -0.57$ in the damped oscillation regime for d = 0.44. The preceding is completely supported by the graphs in Fig. 1, in which the dependences of the time x_m (curve 1) of attainment of the maximum i_m of the discharge-current strength (curve 2), and of the energy of heat release in the load w_m (curve 3) on the damping are presented.

In electric-discharge lasers, the electrical resistance of the electric-discharge plasma $R_c(t)$ that, after plasma formation, decreases with increase in the discharge-current strength, i.e., as the plasma warms up, serves as the load. Consequently, the general solution of Eq. (1) for an organic-dye laser is completely deter-



Fig. 1. Dependence of the time of attainment of the discharge-current strength maximum x_m (1), of the maximum value of the discharge-current strength (2), and of the energy released in the load by the time x_m (3) on the damping in the case of a permanent load.

Fig. 2. Time dependence of the discharge-current strength in the case of an alternate load at d = 1.6 and different values of γ : 1) $\gamma = 0.5$; 2) 1; 3) 1.5; 4) 2; 5) 2.5.

mined by the form of the function $D_c(x)$. In particular, for $D_c(x) = 1/x$ it is the Bessel equation of zero order. Because of this, in the case of such damping, the heat release in the load is described by the Bessel function of the first order that is quasiperiodic in character. The voltage across the capacitor bank and the strength of current in the circuit must be different in sign. When the damping occurs by the exponential law $D_c(x) \sim 1/x^{\gamma}$ at $\gamma = 0$, the general character of evolution of the transient-process parameters must be retained [8], i.e., the general solution of (1) remains quasiperiodic, the zeros of the solution must be spaced almost evenly and, in the limit $x \to \infty$, the intervals between them must approach π . Indeed, as the data of the numerical calculation presented by the curves in Fig. 2 show, for the values $0.5 \le \gamma \le 2.5$ the discharge-current strength $i_c(t)$ is almost periodic in time, i.e., the discharge is essentially quasiperiodic in character. For convenience of comparison with the parameters of the steady damping, the function $D_c(x)$ is selected in the form $D(x) = d(x_m/x)^{\gamma}$. In the discharge circuit with a variable damping, the time of attainment of the first maximum of the current x_m is practically independent of the exponent γ up to $d \approx 2$, which corresponds to the condition D(x) = 1/x, as the data of the numerical solution of Eq. (1), given in Fig. 3a, show. The maximum value of the current strength i_m ceases to depend on d beginning with $\gamma \approx 1.6$, as the data of the numerical solution of (1), presented in Fig. 3b, show. Correspondingly, as is seen from Fig. 3c, the values of the energy released in the load by this time form a plateau on the surface $w_m(d; \gamma) \approx 0.65$ in the ranges $2 \le d$ and $1.1 \le \gamma \le 1.6$. So high a value of the portion of the stored energy, released in the load at the leading edge of the pumping pulse, is due to the nonlinearity of the transient process in the discharge circuit, which, in turn, is determined by the nonlinearity of the equation of discharge-plasma state. It should be noted that if the discharge current is quasiperiodic, as in Fig. 2, in the case where the plasma is cooled at the trailing edge of the current pulse, the equation of its state has a distinctly different form because of the irreversibility of the processes of heating and relaxation in the finite volume. Consequently, the damping in Eq. (1) will be described by a function that, in the general case, is distinct from the exponential function. Thus, the analysis performed is true only for the load representing an electric-discharge plasma whose resistance decreases at the leading edge of the discharge-current pulse. At the same time, the appearance of the maximum on the curve $w_m(d)$ (curve 3) in Fig. 1 obtained for the damped oscillation regime of discharge in the case of a steady load indicates that this energy redistribution is a characteristic property of the transient process and is due to the nonlinearity of this process.



Fig. 3. Dependence of the time of attainment of the first maximum of the discharge-current strength (a), of the maximum value of the discharge-current strength (b), and of the energy released in the load by the time $x_{\rm m}$ (c) on the damping d and the exponent γ in the case of an alternate load.



Fig. 4. Oscillogram of the discharge-current pulse (curve 1) and the electrical resistance of the discharge plasma (curve 2) calculated from the measured values of the current strength and voltage across the resistive load. The scale multiplier along the time axis is 10 μ sec. *R*, Ω ; *t*, μ sec.

It should be particularly emphasized that in the analysis performed the plasma was assumed to be homogeneous, i.e., at every instant the conduction was the same throughout its volume. In the case of contraction of the discharge, the damping in every breakdown channel must be taken into account in its own right. When the process develops in an avalanche-type manner the damping in an individual channel decreases practically exponentially. Since the process of formation of a channel is static in character, and the case in point can be



pulse (c, curve 2). The scale multiplier along the time axis is 10 μ sec. only the statistic of small numbers, the result of calculating the stored-energy portion released in the plasma at the leading edge of the current pulse cannot be predicted, in essence.

Figure 4 shows a typical oscillogram of the discharge-current strength (curves 1 in Figs. 4 and 5) obtained in electrophysical measurements of the parameters of the plasma in a coaxial flashlamp [9-12]. In the same experiments, the homogeneity of the plasma and the time dependences of the inductance voltage (curve 2 in Fig. 5a) and the voltage across the active resistance of the interelectrode spacing (curve 2 in Fig. 5b) were determined. From these data, the time behavior of the electrical resistance of the electric-discharge plasma (curve 2 in Fig. 4) was calculated, beginning with the moment of completion of the formation of a homogeneous hollow plasma column (approximately within 0.3 µsec after commutation) and ending with the moment of attainment of the maximum current. Within the limits of measurement error (~10%), the plasma resistance changes by the hyperbolic law, i.e., $\approx 1/x$ in Eq. (1) (for clarity, the curve R(t) is extended beyond the maximum value of the current strength), the zeros of the discharge current coincide with the same accuracy with the zeros of the Bessel function of the first order, and the current strength is opposite in sign to the voltage across the capacitor bank. Figure 5c shows, for comparison, the oscillograms of the discharge-current strength (curve 1) and of the pumping light pulse (curve 2). The data of these experiments also indicate that d(x) depends significantly on the sort and pressure of the working gas in the discharge space of the lamp. For example, for Xe, at a pressure lower than ~10 torr, at the leading edge of the current pulse, beginning with a time of ≥ 1 µsec and up to a time of ≤ 10 µsec when the maximum current is attained, the electrical resistance of the plasma remains practically constant (Fig. 4), which coincides with the dependence $D(x) \sim 1/x$ for the horizontal branch of the hyperbola. At higher pressures of xenon (in the experiments, as high as 100 torr) the plasma resistance at the leading edge of the current pulse decreases by an exponential or nearly exponential law. In this case, at a pressure of ≤ 10 torr the resistance is practically independent of the working voltage across the capacitor bank, and in the range of 10-100 torr it decreases markedly with increase in the working voltage. However, in all cases, the quasiperiodic character of the discharge is retained. Moreover, the oscillograms in Fig. 5c point to the fact that the leading edges of the pumping and discharge-current pulses are similar.

The quasiperiodic character of change in the characteristics of the transient process suggests that in a certain bounded region of change in the circuit parameters, their mean values can be almost constant. This is qualitatively evidenced by the calculation data of Fig. 3 and by the results of the direct calculation of the time of attainment of the first maximum of the discharge-current strength x_m (Fig. 6a), the maximum value of the discharge-current strength x_m (Fig. 6c) released in the load by this moment, obtained for the values of the exponent $\gamma = 1.2$ (curve 1), $\gamma = 1.4$ (curve 2), $\gamma = 1.6$ (curve 3), $\gamma = 1.8$ (curve 4), and $\gamma = 2.0$ (curve 5). It is apparent that time is not among these parameters, because the averaging over time during the leading edge is meaningless. However, this is possible over other parameters, in particular, over the steady-state damping d that is taken as the initial damping in the calculations. On the seg-



Fig. 6. Dependence of the time of attainment of the first maximum of the discharge-current strength x_m (a), the maximum value of the discharge-current strength i_m (b), and the energy w_m released in the load during the leading edge of the pumping pulse (c) on the initial damping *d* for an alternate load: 1) $\gamma = 1.2$; 2) 1.4; 3) 1.6; 4) 1.8; 5) 2.0.

ment $3 \le d$, the mean value $\overline{x_m} \approx 2.23$ is realized at $\gamma = 1.6$, and $\overline{i_m} \approx -0.61$ is realized on this segment at the same value of $\gamma = 1.6$. At the same time, the energy released in the load at the leading edge behaves differently. It is seen from Fig. 6c that in the range of $\gamma \le 2$ the stored-energy portion converted to heat at the leading edge is bounded above by the value of $w_m \approx 0.65$. This value remains practically constant in the region of $d \ge 2$. Thus, the realization of the parameters found at the values of γ from 1.6 to 2 for $d \ge 3$ makes it possible not only to attain the optimum conditions for the transient process but also to provide its stability.

In the results presented, the constant factor in the damping $D_c(x)$ was considered as the initial damping d for the case of a steady load. This has been done for convenience of the calculation, since this choice does not change the character of the general solution of Eq. (1). Because of this, in principle, d can be replaced by another factor that describes the properties of the medium, in which the electric discharge occurs, for example, by the concentration of molecules of the working gas that determines the pressure in the discharge space prior to breakdown, since the distinctive properties of the plasma, in particular, the degree of its ideality, the mechanism of conduction, and so on, are predominantly dependent on the concentration. The second deciding factor that determines the mentioned properties of the plasma is the plasma temperature, which, in the case of an electric discharge, depends predominantly on the density of the electric energy in the discharge space of the lamp, i.e., eventually, on the mechanism of conduction of the problem is purely technical in character, while the first part calls for additional analysis, since it is determined not only by the electrophysics and physics of the plasma but also, to the same extent, by the properties of the active medium of the laser.

All the necessary calculations were performed with the use of original programs (m-files) in the Matlab 5.2 mathematical-processing medium of MathWorks Inc.

NOTATION

 U_0 , working voltage of the pumping source; $U_{cond}(t)$, instantaneous voltage across the capacitor bank; u_{cond} , dimensionless voltage across the capacitor bank; C_e , capacity of the energy storage; C_c , capacity of the discharge circuit; L_e , inductance of the energy storage; L_c , inductance of the discharge circuit; R_c , active resistance of the energy storage; P_c , instantaneous power of heat release in the active resistance of the load; T_b , brightness temperature; d, damping in the case of a permanent load; Q_c , Q-factor of the discharge circuit; D(x), instantaneous damping in the case of a load that changes by an exponential law; w_c , dimensionless energy of heat release in the active resistance of the load; W_e , energy stored in the energy storage; w_m , maximum energy of heat release in the resistive load; x, dimensionless time; γ , exponent; $J_c(x)$, dimensionless normalized instantaneous strength of current in the discharge circuit; i_m , dimensionless normalized maximum current in the load at the instant x_m ; \bar{x}_m , mean value of the maximum current in the load at the instant \bar{x}_m ; \bar{x}_m , mean value of the dimensionless time; t, time. Subscripts: c, circuit; e, energy storage; m, maximum; cond, condenser; 0, initial conditions.

REFERENCES

- 1. L. G. Nair, Prog. Quant. Electron., 7, 153-268 (1982).
- 2. A. N. Rubinov, Zh. Prikl. Spektrosk., 11, No. 3, 436-443 (1969).
- 3. A. N. Rubinov, Zh. Prikl. Spektrosk., 12, No. 5, 837-844 (1970).
- 4. A. N. Rubinov, Zh. Prikl. Spektrosk., 12, No. 1, 57-64 (1970).
- 5. F. N. Baltakov, B. A. Barikhin, and L. V. Sukhanov, Pis'ma Zh. Eksp. Teor. Fiz., 19, No. 5, 300-302 (1974).
- 6. I. S. Marshak (ed.), Pulsed Light Sources [in Russian], Moscow (1978).
- 7. G. S. Kuchinskii, High-Voltage Pulsed Capacitors [in Russian], Leningrad (1973).
- 8. P. Kaldirola and G. Knopfel (eds.), *Physics of High Energy Density* [Russian translation], Moscow (1974).
- 9. N. Mott and H. S. W. Massey, The Theory of Atomic Collisions [Russian translation], Moscow (1969).
- 10. F. N. Baltakov, B. A. Barikhin, V. G. Kornilov, et al., Zh. Tekh. Fiz. D, 42, No. 7, 1459-1461 (1972).
- 11. F. N. Baltakov and B. A. Barikhin, Kvant. Elektron., 2, No. 4, 822-826 (1975).
- 12. B. A. Barikhin, B. S. Makaev, L. V. Sukhanov, and A. I. Pavlovskii, *Kvant. Elektron.*, **3**, No. 6, 1211-1216 (1976).